Estimating the Size of an Average Personal Network and of an Event Subpopulation: Some Empirical Results

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A compelling problem in a population is to estimate the number of people the average person knows. A consequential related problem is to estimate the size of important subpopulations. A random sample of a population is asked whether they know anyone in a given subpopulation of size \( c \), thus yielding an estimate of the probability that this occurs in the population. Using an equal likelihood probability model, this leads to a lower bound estimate for \( c \), the average number of people a person in the population knows. When the number of people a person knows has a binomial distribution over the population this value is an estimate for \( c \) itself. Here we test this method on data from Mexico City, where a large...
random sample of people was asked whether they knew anyone in each of several different subpopulations in Mexico City of known and unknown sizes. We develop procedures for obtaining various bounds and estimates for \( c \) and determine some of the respondents' attributes on which variation in probability of knowing someone in a subpopulation and variation in personal network size seem to depend. We apply these to the estimation of \( c \) for rape victims in Mexico City and the estimation of \( c \) from data on AIDS victims in the United States. © 1991 Academic Press, Inc.

An important but vexing problem in social network analysis has been to determine in a population the number of people a person knows, i.e., his/her personal network size, and the mean, range, and distribution of this variable in the population as a whole (cf. de Sola Pool and Kochen, 1978). These data have heretofore defied successful investigation. In an earlier paper (Bernard, Johnsen, Killworth, and Robinson, 1989), we presented a probabilistic method for estimating the average size of a personal network and the size of an event subpopulation in a given total population. We applied it to a first small random data sample of size 400 from the population of the Federal District of Mexico City in order to relate various proposed sizes of the subpopulation of victims of the 1985 Mexico City earthquake to the average personal network size in the Federal District. We give here and in the next two sections a brief summary of this method and these first results. We then apply the method to the data from a second larger random sample from the population of Mexico City proper in order to obtain estimates of the size of the average personal network from various known event subpopulation sizes, and then use this information to estimate the size of an unknown event subpopulation and to compare against results for a known event subpopulation in the U.S.

Consider a population \( T \), of size \( t \), having a subpopulation \( E \), of size \( e > 0, e \ll t \), which is the subgroup of \( T \) associated with some attribute or event. For each member \( u \) of \( T-E \) let \( k(u) \) denote the number of people in \( T \) that \( u \) "knows." Here "\( u \) knows \( v \)" means that \( u \) knows \( v \) personally, in that \( u \) knows \( v \) by name, knows where \( v \) lives, knows \( v \)'s occupation, and that \( v \) knows the same about \( u \). The people in \( T \) whom \( u \) knows will be called the personal network of \( u \), denoted by \( K(u) \).

We allow \( k(u) \) to vary with \( u \) over \( T-E \) and to take its values on a finite interval of nonnegative integers \([n_0, n_0 + n]\), where \( n > 0 \). Regarding average personal network size, we give some results on the general case and the case where \( k(u) \) has a binomial distribution. We then address the question of estimating event subpopulation size.

Now, we need to make a fundamental assumption, either about the distribution of the members in the various personal networks \( K(u) \) or about the distribution of the members of \( E \), as follows:

A. For a random member \( u \) of \( T-E \), all subsets of \( T-\{u\} \) of size \( k(u) \) are equally likely to have been the subset \( K(u) \) known by \( u \).

B. All subsets of \( T \) of size \( c \) were equally likely to have been the subpopulation \( E \).

In some situations (but possibly not some of those discussed in this paper, e.g., the Mexico City earthquake) version B seems plausible. In the case of the earthquake, if all of the downtown buildings in a city were similar in level of earthquake survivability and all socioeconomic strata of the population were randomly represented in the downtown population when an earthquake occurred centered downtown, etc., then this assumption may not be a bad one. Version A is equivalent to assuming that for a random \( u \) in \( T-E \) the probability that any particular member of \( K(u) \) is in \( E \) is just the relative size of \( E \) in \( T \), \( e/t \), when \( e \) is very small compared to \( t \).

### AVERAGE PERSONAL NETWORK SIZE

For the general case, where the distribution of \( k(u) \) for \( u \) in \( T-E \) is unspecified, we have the following results (Bernard et al., 1989). We let \( p \) denote the proportion of the members of \( T-E \) who know someone in \( E \), \( ln \) is the natural logarithm, \( g \) is a certain real number in the range \( 1 < g \leq \max[k(u) \mid u \in T-E] \), and \( e = e/t \).

**LEMMA 1.** Under either of the assumptions A or B, the value

\[
\alpha = \ln(1 - p)/\ln(1 - e/(1 - g)) \approx \ln(1 - p)/\ln(1 - e),
\]

(1)
determined by the values of \( e, p, \) and \( t \) and the distribution of \( k(u) \), must lie within the range of values \([n_0, n_0 + n]\). The right hand numerical approximation in (1) is excellent when \( n_0 + n \) is very small compared to \( t \).

**THEOREM 2.** Under either assumption A or B and for any probability distribution of the values \( k(u) \) on the integer interval \([n_0, n_0 + n]\), the value \( \alpha \) and the average value \( c \) of the personal network sizes \( k(u) \) must satisfy the inequalities

\[
n_0 \leq \alpha \leq c \leq n_0 + n.
\]

(2)

For the one-point distribution, where \( n = 0 \), all three inequalities in (2) are equalities. For a distribution with at least two points, where \( n > 0 \), all three inequalities are strict inequalities <. 

With an empirical estimate for \( p \) and under either of the distribution assumptions A or B we can estimate \( \alpha \), and frequently \( c \), by the right side of (1). For, if \( \alpha_1, \alpha_2, \ldots, \alpha_5 \) are values corresponding to different event subpopulations \( E_1, E_2, \ldots, E_5 \) we should have

\[
c \geq \max_{i \leq 5} \alpha_i.
\]

(3)
random sample of people was asked whether they knew anyone in each of several different subpopulations in Mexico City of known and unknown sizes. We develop procedures for obtaining various bounds and estimates for $c$ and determine some of the respondents' attributes on which variation in probability of knowing someone in a subpopulation and variation in personal network size seem to depend. We apply these to the estimation of $c$ for rape victims in Mexico City and the estimation of $c$ from data on AIDS victims in the United States. © 1991 Academic Press, Inc.

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Consider a population $T$, of size $t$, having a subpopulation $E$, of size $e > 0$, $e << t$, which is the subgroup of $T$ associated with some attribute or event. For each member $u$ of $T-E$ let $k(u)$ denote the number of people in $T$ that $u$ "knows." Here "$u$ knows $v$" means that $u$ knows $v$ personally, in that $u$ knows $v$ by name, knows where $v$ lives, knows $v$'s occupation, and that $v$ knows the same about $u$. The people in $T$ whom $u$ knows will be called the personal network of $u$, denoted by $K(u)$.

We allow $k(u)$ to vary with $u$ over $T-E$ and to take its values on a finite interval of nonnegative integers $[n_0, n_0 + n]$, where $n > 0$. Regarding average personal network size, we give some results on the general case and the case where $k(u)$ has a binomial distribution. We then address the question of estimating event subpopulation size.

Now, we need to make a fundamental assumption, either about the distribution of the members in the various personal networks $K(u)$ or about the distribution of the members of $E$, as follows:

**A.** For a random member $u$ of $T-E$, all subsets of $T-\{u\}$ of size $k(u)$ are equally likely to have been the subset $K(u)$ known by $u$.

**B.** All subsets of $T$ of size $e$ were equally likely to have been the subpopulation $E$.

In some situations (but possibly not some of those discussed in this paper, e.g., the Mexico City earthquake) version B seems plausible. In the case of the earthquake, if all of the downtown buildings in a city were similar in level of earthquake survivability and all socioeconomic strata of the population were randomly represented in the downtown population when an earthquake occurred centered downtown, etc., then this assumption may not be a bad one. Version A is equivalent to assuming that for a random $u$ in $T-E$ the probability that any particular member of $K(u)$ is in $E$ is just the relative size of $E$ in $T$, $e/t$, when $e$ is very small compared to $t$.

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**Lemma 1.** Under either of the assumptions A or B, the value

$$\alpha = \ln(1 - p)/\ln[1 - e/(t - g)] \approx \ln(1 - p)/\ln(1 - e).$$

(1)

is determined by the values of $e$, $p$, and $t$ and the distribution of $k(u)$, must lie within the range of values $[n_0, n_0 + n]$. The right hand numerical approximation in (1) is excellent when $n_0 + n$ is very small compared to $t$.

**Theorem 2.** Under either assumption A or B and for any probability distribution of the values $k(u)$ on the integer interval $[n_0, n_0 + n]$, the value $\alpha$ and the average value $c$ of the personal network sizes $k(u)$ must satisfy the inequalities

$$n_0 \leq \alpha \leq c \leq n_0 + n.$$

(2)

For the one point distribution, where $n = 0$, all three inequalities in (2) are equalities. For a distribution with at least two points, where $n > 0$, all three inequalities are strict inequalities $<$. With an empirical estimate for $p$ and under either of the distribution assumptions A or B we can estimate $\alpha$, and frequently $c$, by the right side of (1). For, if $\alpha_1, \alpha_2, \ldots, \alpha_s$ are values corresponding to different event subpopulations $E_1, E_2, \ldots, E_s$, we should have

$$c \approx \max_{1 \leq i \leq s} \alpha_i.$$

(3)
Our first Mexico City earthquake sample of 400 random respondents did not meet our statistical requirements; however, it was tantalizing to use the data from this sample to find \( \alpha \) as a lower bound estimate for \( c \). With \( p = 91/400 = .2275 \), we computed \( \alpha \) for the different proposed death counts \( e \) to the nearest integer in Table 1 (taking \( g = 0 \)). The right hand approximation in (1) is correct to within .1% here, assuming \( n_0 + n \leq 10000 \).

We now consider the special case where \( k(u) \) is assumed to have a binomial distribution over its range \([n_0, n_0 + n]\). Here \( n \) may be viewed as the number of opportunities or encounters (the “trials” of the binomial distribution) that \( u \) has with other members of \( T \), over and above a fixed set of \( n_0 \) members whom \( u \) already knows, each of which has a fixed probability of resulting in \( u \) knowing \( v \) (i.e., resulting in “success”). For this case we have

\[
c = \alpha
\]  

Thus, the values of \( c \) for the Mexico City earthquake data when \( k(u) \) has a binomial distribution over its range are virtually the same as those given in Table 1, and the approximate equality (4) is practically independent of the value of the fixed trial success probability.

### EVENT SUBPOPULATION SIZE

For the probability distribution of \( k(u) \) with \( \hat{p} = 1 - p, \hat{e} = 1 - e \), and \( q_m = P(k(u) = n_0 + m), m = 0, 1, \ldots, n \), we obtained (Bernard et al., 1989) that

\[
\hat{p} = \sum_{m=0}^{n} q_m \hat{e}^{n_0 + m}.
\]  

Since \( \hat{e} > 0 \) and \( q_m \geq 0 \) for all \( m = 0, 1, \ldots, n \) with some \( q_m > 0 \), the derivative of \( \hat{p} \) with respect to \( \hat{e} \) is positive, whence \( \hat{p} \) is an increasing function of \( \hat{e} \) and so \( p \) is an increasing function of \( e \). Thus, if there are event subpopulations \( E_1, E_2, \ldots, E_n \), ordered so their corresponding \( e_i \) values satisfy

\[
e_0 < e_1 < e_2 < \cdots < e_i,
\]  

then we also have for their corresponding \( p_i \) values

\[
p_0 = 0 < p_1 < p_2 < \cdots < p_s,
\]  

where \( \hat{e}_0 = 1 - e_0 = 1 \) and \( \hat{p}_0 = 1 - p_0 = 1 \) also satisfy (5).

Now let \( E_x \) be a new event subpopulation of unknown size \( e_x \) and unknown relative size \( e_x \), for which the probability \( p_x \) that a random \( u \) in \( T - E_x \) knows anyone in \( E_x \) satisfies in (7)

\[
p_{k-1} < p_x < p_k, \quad \text{for some } k, 1 \leq k \leq s.
\]  

Then, from (6) we have

\[
e_{k-1} < e_x < e_k.
\]  

Thus, if our probability model is reasonably close to correct then, given a sufficiently broad range of \( e_x, p_x \) pairs from previous event subpopulations, we should be able to bound the size of the new event subpopulation between successive values

\[
e_{k-1} < e_x < e_k, \quad \text{for some } k, 1 \leq k \leq s.
\]  

Clearly, if (8) is true but not (9) and (10) then either the data values \( e_i, p_i, 1 \leq i \leq s \), are poor or the original probability model is not completely correct. Thus, if the data are believed to be good we have a negative criterion for the full validity of the underlying probability model.

Assuming in this model that \( \hat{p} \) is a differentiable function of \( \hat{e} \), we have from (5) that

\[
d\hat{p}/d\hat{e} |_{\hat{e}=1} = \sum_{m=0}^{n} (n_0 + m) q_m \hat{e}^{n_0 + m} |_{\hat{e}=1}
\]  

\[
= \sum_{m=0}^{n} (n_0 + m) q_m = c.
\]

Thus, for a fairly large value for \( c \) (at least 211 by Table 1) and for \( \hat{e} \) less than but very close to 1, we see that large changes in \( \hat{p} \) correspond to small changes in \( \hat{e} \). This indicates that whatever the size of the bound within which \( p_x \) sits in (8), the corresponding size of the bound for the approximation of \( e_x \) in (9) will be considerably smaller.

Now suppose that for a fixed population \( T \) (more precisely, \( T - E \) for which \( e \) is very small relative to \( t \)) we know the average personal network size \( c \). From (1) we derive the relation

\[
1 - p = (1 - e)^n,
\]  

where \( \alpha \) is a function of \( e \) and \( p \) and, hence, need not be constant over different pairs \( e, p \). Now, by (2), we have for \( 0 < e < 1 \) that

\[
(1 - e)^n \approx (1 - e)^c
\]  

\[12 

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Our first Mexico City earthquake sample of 400 random respondents did not meet our statistical requirements; however, it was tantalizing to use the data from this sample to find $c$ as a lower bound estimate for $c$. With $p = 91/400 = 0.2275$, we computed $\alpha$ for the different proposed death counts $e$ to the nearest integer in Table 1 (taking $g = 0$). The right hand approximation in (1) is correct to within .1% here, assuming $n_0 + n \leq 10000$.

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Thus, the values of $c$ for the Mexico City earthquake data when $k(u)$ has a binomial distribution over its range are virtually the same as those given in Table 1, and the approximate equality (4) is practically independent of the value of the fixed trial success probability.

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For the probability distribution of $k(u)$ with $\tilde{\alpha} = 1 - \tilde{e}$, and $q_m = P(k(u) = n_0 + m)$, $m = 0, 1, \ldots, n$, we obtained (Bernard et al., 1989) that

$$\tilde{p} = \sum_{m=0}^{n} q_m \tilde{e}_0^{n_0 + m}.$$

Since $\tilde{e}_0 > 0$ and $q_m > 0$ for all $m = 0, 1, \ldots, n$ with some $q_m > 0$, the derivative of $\tilde{p}$ with respect to $\tilde{e}$ is positive, whence $\tilde{p}$ is an increasing function of $\tilde{e}$ and so $\tilde{p}$ is an increasing function of $e$. Thus, if there are event subpopulations $E_1, E_2, \ldots, E_n$, ordered so their corresponding $e_i$ values satisfy

$$e_0 = 0 < e_1 < e_2 \ldots < e_i,$$

then we also have for their corresponding $p_i$ values

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$$p_{k-1} < p_k < p_k$$

for some $k$, $1 \leq k \leq x$.

Then, from (6) we have

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Thus, if our probability model is reasonably close to correct then, given a sufficiently broad range of $e_i$, $p_i$ pairs from previous event subpopulations, we should be able to bound the size of the new event subpopulation between successive values

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Clearly, if (8) is true but not (9) and (10) then either the data values $e_i$, $p_i$, $1 \leq i \leq x$, are poor or the original probability model is not completely correct. Thus, if the data are believed to be good we have a negative criterion for the full validity of the underlying probability model.

Assuming in this model that $\tilde{p}$ is a differentiable function of $\tilde{e}$, we have from (5) that

$$d\tilde{p}/d\tilde{e} \bigg|_{\tilde{e}=1} = \sum_{m=0}^{n} (n_0 + m) q_m \tilde{e}_0^{n_0 + m} \bigg|_{\tilde{e}=1}$$

$$= \sum_{m=0}^{n} (n_0 + m) q_m = c.$$

Thus, for a fairly large value for $c$ (at least 211 by Table 1) and for $\tilde{e}$ less than but very close to 1, we see that large changes in $\tilde{p}$ correspond to small changes in $\tilde{e}$. This indicates that whatever the size of the bound within which $p_x$ sits in (8), the corresponding size of the bound for the approximation of $e_x$ in (9) will be considerably smaller.

Now suppose that for a fixed population $T$ (more precisely, $T - E$ for which $e$ is very small relative to $t$) we know the average personal network size $c$. From (1) we derive the relation

$$1 - p = (1 - e)^n,$$

where $\alpha$ is a function of $e$ and $p$ and, hence, need not be constant over different pairs $e$, $p$. Now, by (2), we have for $0 < e < 1$ that

$$(1 - e)^n \geq (1 - e)^\alpha.$$
or, by (12),
\[ \varepsilon \geq 1 - (1 - p)^{\frac{1}{10}} = \bar{\varepsilon}. \] (14)

Thus, for \( E \) of unknown relative size \( \varepsilon \), with an accurately estimated probability \( p \), of a person in \( T-E \), knowing someone in \( E \), we have
\[ \bar{\varepsilon} = 1 - (1 - p)^{\frac{1}{10}} \leq \varepsilon, \] (15)

which yields a lower bound approximation \( \bar{\varepsilon} \) to the true value \( \varepsilon \). Here, the closer \( \varepsilon \) is to \( \alpha \), or \( \epsilon \), is to 0, the better the approximation \( \bar{\varepsilon} \) is to \( \varepsilon \). The latter implication corresponds to the fact that the closer \( \bar{\varepsilon} \) is to 1 the better the approximation of \( \bar{\varepsilon} \) by \( \bar{\varepsilon} \).

As an example, from the first sample data for the Mexico City earthquake, with event subpopulation \( E \) of assumed size \( e = 7000 \) and probability \( p = .2275 \), we determined that \( \varepsilon = 664 \). Now suppose for a new event subpopulation \( E \), we obtain \( p = .1986 \). Then the size \( e \), is bounded by \( e < 7000 \) and underestimated by \( \bar{\varepsilon} = \bar{\varepsilon}; t = [1 - (0.8014)^{\frac{1}{10}}] \cdot (18,000,000) = 6000.67 \), so 6001 \( \leq \varepsilon < 7000 \). Now, from (5), the graph of \( \hat{\rho} \) as a function of \( \hat{\varepsilon} \) is increasing and concave upward; hence, we can do a linear interpolation between the points (\( \hat{\varepsilon}, \hat{\rho} \)) = (1, 1) and (\( \hat{\varepsilon}, \hat{\rho} \)) = (0.99961111, .7725) to obtain the tighter upper bound
\[ e \leq 6110, \] whence \( 6001 \leq \varepsilon < 6110 \).

APPLICATION TO A LARGE DATA SAMPLE FROM MEXICO CITY

In a later second survey we obtained data from a larger random sample of 2260 from Mexico City proper (\( t = 10,700,000 \)) in the hope of establishing a set of reference value pairs \( (p, e) \), against which to compare new value pairs \( (p, e) \) above the data for the six reference event subpopulations, with the 95% confidence ranges for \( p \) and corresponding ranges for \( \alpha \), are given in Table 2.

It is clear from this table that the monotonicity property of the model given by (8) and (9) does not hold, which indicates that either the data are inaccurate, the data are reasonably accurate but not very precise, or the model is not valid in its simple form for different subpopulations (or possibly a combination of either the first or second and the third).

By the nature of the survey and the data obtained, the first alternative (including its combination with the third) appears untenable. But, before ruling out the model in its simple form, we investigate the data to see whether they are reasonably accurate but not very precise. This suggests analyzing the data at the aggregate level to gain precision and, it is hoped, detect a “signal” amidst the “noise.” Since the simple model (1) with the assumption of an approximately binomial distribution of \( k(u) \) for \( u \) in \( T \) (or \( T-E \) produces an approximately constant \( c = \alpha = \ln \hat{\rho} / \ln \hat{\varepsilon} \), which says that \( \ln \hat{\rho} \) and \( \ln \hat{\varepsilon} \) vary linearly with respect to each other, we attempt to estimate \( c \) (via \( \alpha \)) by least squares linear regression of each variable against the other. Now, there are four ways to do this, namely,

\[ \ln \hat{\rho} = \alpha \ln \hat{\varepsilon}, \] (16)
\[ \ln \hat{\rho} = \alpha \ln \hat{\varepsilon} + \ln \beta, \] (17)
\[ \ln \hat{\varepsilon} = \alpha^{-1} \ln \hat{\rho}, \] (18)
\[ \ln \hat{\varepsilon} = \alpha^{-1} \ln \hat{\rho} + \ln \gamma, \] (19)

where \( \ln \beta \) and \( \ln \gamma \) are included in the unconstrained regressions. The unconstrained regressions are included to see how well their regression lines fall naturally into place on the basis of the empirical data alone without imposition of the intercept 0. We obtain the following results, with \( \alpha \) to the nearest integer, for the data in Table 2:

\[ \alpha = 196, \] (16.r)
\[ \alpha = 56, \quad \beta = .7761, \] (17.r)
\[ \alpha = 274, \] (18.r)
\[ \alpha = 221, \quad \gamma = 1.0003. \] (19.r)

The values of \( \alpha \) in (16.r) and (18.r) for the constrained model yield the range 235 \( \pm \) 39, which includes the \( \alpha \) values for Bus Drivers and for line (19). The data point for Bus Drivers almost lies on all three lines (17), (18), and (19). Except for the data point for Quake Victims, the other data points do lie very close to any of these lines. Since \( \ln .7761 = -0.253 \) and \( \ln 1.0003 = .0003 \), line (17) suggests some problem with the data or the model, whereas line (19) does not. We may also determine the best fit lines (16) and (17) in the sense of least squares distance of the data points from the lines (lines (18) and (19) are then, respectively, equivalent to (16) and (17)). For these we obtain the following results from the data in Table 2:
<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>e</th>
<th>Range of p</th>
<th>(\alpha)</th>
<th>Range of (\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>30,426</td>
<td>.3889 ± .0201</td>
<td>173</td>
<td>162–185</td>
</tr>
<tr>
<td>Mailmen</td>
<td>14,728</td>
<td>.1473 ± .0146</td>
<td>116</td>
<td>103–128</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>11,696</td>
<td>.2571 ± .0180</td>
<td>272</td>
<td>250–294</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>10,000</td>
<td>.2668 ± .0182</td>
<td>332</td>
<td>306–359</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>4,013</td>
<td>.2619 ± .0181</td>
<td>810</td>
<td>745–876</td>
</tr>
<tr>
<td>Priests</td>
<td>1,595</td>
<td>.2854 ± .0186</td>
<td>2254</td>
<td>2082–2431</td>
</tr>
</tbody>
</table>

or, by (12),

\[
\varepsilon \geq 1 - (1 - p)^{\frac{1}{e}} = \tilde{\varepsilon}. \tag{14}
\]

Thus, for \(E_x\) of unknown relative size \(e_x\), with an accurately estimated probability \(p_x\) of a person in \(T-E_x\), knowing someone in \(E_x\), we have

\[
\tilde{\varepsilon}_x = 1 - (1 - p_x)^{\frac{1}{e_x}} \leq \varepsilon_x, \tag{15}
\]

which yields a lower bound approximation \(\tilde{\varepsilon}_x\) to the true value \(\varepsilon_x\). Here, the closer \(c\) is to \(\alpha\), or \(\varepsilon_x\) is to 0, the better the approximation \(\tilde{\varepsilon}_x\) is to \(e_x\). The latter implication corresponds to the fact that the closer \(\tilde{\varepsilon}\) is to 1 the better the approximation of \(\tilde{\varepsilon}\) by \(\tilde{\varepsilon}_x\).

As an example, from the first sample data for the Mexico City earthquake with event subpopulation \(E\) of assumed size \(e = 7000\) and probability \(p = .2275\), suppose we determined that \(e \approx 664\). Now suppose for a new event subpopulation \(E_x\) we obtain \(p_x = .1986\). Then the size \(e_x\) is bounded by \(e_x < 7000\) and underestimated by \(\tilde{\varepsilon}_x = \tilde{\varepsilon}_x; t = [1 - (.8014)^{100}] \cdot (18,000,000) = 6000.67\), so \(6001 \leq e_x < 7000\). Now, from (5), the graph of \(\tilde{\varepsilon}\) as a function of \(\tilde{\varepsilon}\) is increasing and concave upward; hence, we can do a linear interpolation between the points \((\tilde{\varepsilon}_1, \tilde{\varepsilon}_1) = (1, 1)\) and \((\tilde{\varepsilon}_2, \tilde{\varepsilon}_2) = (.99961111 \ldots, .7725)\) to obtain the tighter upper bound \(e_x \leq 6110\), whence \(6001 \leq e_x \leq 6110\).

**APPLICATION TO A LARGE DATA SAMPLE FROM MEXICO CITY**

In a later second survey we obtained data from a larger random sample of 2260 from Mexico City proper \((t = 10,700,000)\) in the hope of establishing a set of reference value pairs \((p_x, e_x)\) against which to compare new value pairs \((p_x, e_x)\) according to (8) and (9) above. The data for the six reference event subpopulations, with the 95% confidence ranges for \(p\) and corresponding ranges for \(\alpha\), are given in Table 2.

It is clear from this table that the monotonicity property of the model given by (8) and (9) does not hold, which indicates that either the data are inaccurate, the data are reasonably accurate but not very precise, or the model is not valid in its simple form for different subpopulations (or possibly a combination of either the first or second and the third).

By the nature of the survey and the data obtained, the first alternative (including its combination with the third) appears untenable. But, before ruling out the model in its simple form, we investigate the data to see whether they are reasonably accurate but not very precise. This suggests analyzing the data at the aggregate level to gain precision and, in it is hoped, detect a "signal" amidst the "noise." Since the simple model (1) with the assumption of an approximately binomial distribution of \(k(u)\) for \(u\) in \(T\) (or \(T-E\)) produces an approximately constant \(c = \alpha = \ln \tilde{\beta} / \ln \tilde{\varepsilon}\), which says that \(\ln \tilde{\beta}\) and \(\ln \tilde{\varepsilon}\) vary linearly with respect to each other, we attempt to estimate \(c\) (via \(\alpha\)) by least squares linear regression of each variable against the other. Now, there are four ways to do this, namely,

\[
\ln \tilde{\beta} = \alpha \cdot \ln \tilde{\varepsilon}, \tag{16}
\]

\[
\ln \tilde{\beta} = \alpha \cdot \ln \tilde{\varepsilon} + \ln \beta, \tag{17}
\]

\[
\ln \tilde{\varepsilon} = \alpha^{-1} \cdot \ln \tilde{\beta} \tag{18}
\]

\[
\ln \tilde{\varepsilon} = \alpha^{-1} \cdot \ln \tilde{\beta} + \ln \gamma, \tag{19}
\]

where \(\ln \beta\) and \(\ln \gamma\) are included in the unconstrained regressions. The unconstrained regressions are included to see how well their regression lines fall naturally into place on the basis of the empirical data alone without imposition of the intercept 0. We obtain the following results, with \(\alpha\) to the nearest integer, for the data in Table 2:

\[
\alpha = 196, \tag{16.r}
\]

\[
\alpha = 56, \quad \beta = .7761, \tag{17.r}
\]

\[
\alpha = 274, \tag{18.r}
\]

\[
\alpha = 221, \quad \gamma = 1.0003. \tag{19.r}
\]

The values of \(\alpha\) in (16.r) and (18.r) for the constrained model yield the range 235 ± 39, which includes the \(\alpha\) values for Bus Drivers and for line (19). The data point for Bus Drivers almost lies on all three lines (17, 18), and (19). Except for the data point for Quake Victims, the other data points do not lie very close to any of these lines. Since \(\ln .7761 = -.253\) and \(\ln 1.0003 = .0003\), line (17) suggests some problem with the data or the model, whereas line (19) does not. We may also determine the best fit lines (16) and (17) in the sense of least squares distance of the data points from the lines (lines (18) and (19) are then, respectively, equivalent to (16) and (17)). For these we obtain the following results from the data in Table 2:
TABLE 3
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Age in Years

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>&lt;20</th>
<th>20–34</th>
<th>35–49</th>
<th>50–65</th>
<th>&gt;65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.3171</td>
<td>.3756</td>
<td>.4484</td>
<td>.4921</td>
<td>.4000</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.1111</td>
<td>.1503</td>
<td>.1659</td>
<td>.1746</td>
<td>.1143</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.2685</td>
<td>.2599</td>
<td>.2646</td>
<td>.2063</td>
<td>.2000</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.2824</td>
<td>.2789</td>
<td>.2399</td>
<td>.2275</td>
<td>.2286</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.2847</td>
<td>.2798</td>
<td>.2399</td>
<td>.1852</td>
<td>.0857</td>
</tr>
<tr>
<td>Priests</td>
<td>.2269</td>
<td>.2720</td>
<td>.3363</td>
<td>.3598</td>
<td>.4000</td>
</tr>
</tbody>
</table>

\[ \alpha = 274, \quad (16.s) \]

\[ \alpha = 221, \quad \beta = .9356. \quad (17.s) \]

Since In .9356 = −.0666 and these best fit lines (which treat the variables symmetrically) are virtually the same as (18.r) and (19.r), respectively, this again supports the corresponding \( \alpha \) values 248 ± 27. Both (17.s) and (19.r) suggest that lack of precision could be the culprit in the scatter of the data. With an approximately binomial distribution for \( k(u) \) and data of sufficiently high precision, the simple model may be adequate for estimating \( e \) from \( c \) and vice versa.

For each of the six reference subpopulations the sample (and hence also the total population) was partitioned into subclasses according to (i) zone of survey interview, whether socioeconomically lower, middle, or upper class; (ii) age of respondent, whether <20, 20–34, 35–49, 50–65, or >65 years; (iii) highest education level attained by respondent, whether <4, 4–6, 7–12, 13–16, or >16 years; (iv) socioeconomic class of respondent, whether upper, middle, or lower class; (v) occupation of respondent, whether working at home (home), out of home (oohm), retired (red), unemployed (unem), students (stud), or a residual class (rslld); and (vi) respondent's reporting of how many people he/she believes to be in

TABLE 4
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Education in Years

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>&lt;4</th>
<th>4–6</th>
<th>7–12</th>
<th>13–16</th>
<th>&gt;16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.2539</td>
<td>.2769</td>
<td>.3735</td>
<td>.5146</td>
<td>.7333</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.0933</td>
<td>.1076</td>
<td>.1549</td>
<td>.1683</td>
<td>.2519</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.3164</td>
<td>.2749</td>
<td>.2833</td>
<td>.1756</td>
<td>.1556</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.1554</td>
<td>.2032</td>
<td>.2853</td>
<td>.3244</td>
<td>.3481</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.1399</td>
<td>.1096</td>
<td>.2814</td>
<td>.3976</td>
<td>.4444</td>
</tr>
<tr>
<td>Priests</td>
<td>.2176</td>
<td>.2470</td>
<td>.2892</td>
<td>.3146</td>
<td>.4074</td>
</tr>
</tbody>
</table>

TABLE 5
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Socioeconomic Class

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>Lower</th>
<th>Middle</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.2696</td>
<td>.4644</td>
<td>.6323</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.1239</td>
<td>.1743</td>
<td>.1097</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.3211</td>
<td>.2245</td>
<td>.0710</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.2200</td>
<td>.3066</td>
<td>.2903</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.1606</td>
<td>.3203</td>
<td>.5097</td>
</tr>
<tr>
<td>Priests</td>
<td>.2250</td>
<td>.3285</td>
<td>.3742</td>
</tr>
</tbody>
</table>

his/her personal network, whether ≤100, 100–500, 500–1000, 1000–1500, >1500, or no answer. We present the \( p \) values for the subsamples determined by age, education, socioeconomic class, occupation, and number believed known in Tables 3, 4, 5, 6, and 7. Since the full sample was not a quota sample for the different subclasses of \( T \) the subsample sizes varied considerably, from 16 in How Many: No Answer to 1158 in Age: 20–34.

We note that there is a great deal of monotonicity in this data, either increasing or decreasing values of \( p \) with increasing values of the table variable when the table variable has a natural ordering. For example, the data for Priests show that \( p \) increases without exception with increasing age, education, and socioeconomic class and almost without exception with increasing believed personal network size. For Doctors, \( p \) increases without exception with increasing education, socioeconomic class, and believed personal network size and almost without exception with increasing age. A similar but weaker version of this occurs for Mailmen.

With regard to all six reference subpopulations, \( p \) increases with increasing believed personal network size without exception for Doctors and Quake Victims and almost without exception for Mailmen, Bus Drivers, TV Repairmen, and Priests. Thus, believed personal network size behaves like actual personal network size should behave with respect to
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<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>&lt;20</th>
<th>20–34</th>
<th>35–49</th>
<th>50–65</th>
<th>&gt;65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.3171</td>
<td>.3756</td>
<td>.4484</td>
<td>.4921</td>
<td>.4001</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.1111</td>
<td>.1503</td>
<td>.1659</td>
<td>.1746</td>
<td>.1143</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.2685</td>
<td>.2599</td>
<td>.2646</td>
<td>.2063</td>
<td>.2000</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.2824</td>
<td>.2789</td>
<td>.2399</td>
<td>.2275</td>
<td>.2286</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.2847</td>
<td>.2798</td>
<td>.2399</td>
<td>.1852</td>
<td>.0857</td>
</tr>
<tr>
<td>Priests</td>
<td>.2269</td>
<td>.2720</td>
<td>.3363</td>
<td>.3598</td>
<td>.4000</td>
</tr>
</tbody>
</table>

\[ \alpha = 274, \quad (16.8) \]
\[ \alpha = 221, \quad \beta = .9356. \quad (17.8) \]

Since ln .9356 = −.0666 and these best fit lines (which treat the variables symmetrically) are virtually the same as (18.4) and (19.4), respectively, this again supports the corresponding \( \alpha \) values 248 ± 27. Both (17.8) and (19.4) suggest that lack of precision could be the culprit in the scatter of the data. With an approximately binomial distribution for \( k(u) \) and data of sufficiently high precision, the simple model may be adequate for estimating e from e and vice versa.

For each of the six reference subpopulations the sample (and hence also the total population) was partitioned into subclasses according to (i) zone of survey interview, whether socioeconomically lower, middle, or upper class; (ii) age of respondent, whether <20, 20–34, 35–49, 50–65, or >65 years; (iii) highest education level attained by respondent, whether 4, 4–6, 7–12, 13–16, or >16 years; (iv) socioeconomic class of respondent, whether upper, middle, or lower class; (v) occupation of respondent, whether working at home (home), out of home (oohm), retired (retd), unemployed (unem), students (stud), or a residual class (rsdl); and (vi) respondent’s reporting of how many people he/she believes to be in

TABLE 4
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Education in Years

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>&lt;4</th>
<th>4–6</th>
<th>7–12</th>
<th>13–16</th>
<th>&gt;16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.2539</td>
<td>.2769</td>
<td>.3735</td>
<td>.5146</td>
<td>.7333</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.0933</td>
<td>.1076</td>
<td>.1549</td>
<td>.1683</td>
<td>.2519</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.3161</td>
<td>.2749</td>
<td>.2833</td>
<td>.1756</td>
<td>.1556</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.1554</td>
<td>.2032</td>
<td>.2853</td>
<td>.3244</td>
<td>.3481</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.1399</td>
<td>.1096</td>
<td>.2814</td>
<td>.3976</td>
<td>.4444</td>
</tr>
<tr>
<td>Priests</td>
<td>.2176</td>
<td>.2470</td>
<td>.2892</td>
<td>.3146</td>
<td>.4074</td>
</tr>
</tbody>
</table>

TABLE 5
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Socioeconomic Class

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>Lower</th>
<th>Middle</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.2696</td>
<td>.4644</td>
<td>.6323</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.1239</td>
<td>.1743</td>
<td>.1097</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.3211</td>
<td>.2245</td>
<td>.0710</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.2200</td>
<td>.3066</td>
<td>.2903</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.1606</td>
<td>.3203</td>
<td>.5097</td>
</tr>
<tr>
<td>Priests</td>
<td>.2250</td>
<td>.3285</td>
<td>.3742</td>
</tr>
</tbody>
</table>

his/her personal network, whether ≤100, 100–500, 500–1000, 1000–1500, >1500, or no answer. We present the \( p \) values for the subsamples determined by age, education, socioeconomic class, occupation, and number believed known in Tables 3, 4, 5, 6, and 7. Since the full sample was not a quota sample for the different subclasses of \( T \) the subsample sizes varied considerably, from 16 in How Many: No Answer to 1158 in Age: 20–34.

We note that there is a great deal of monotonicity in this data, either increasing or decreasing values of \( p \) with increasing values of the table variable when the table variable has a natural ordering. For example, the data for Priests show that \( p \) increases without exception with increasing age, education, and socioeconomic class and almost without exception with increasing believed personal network size. For Doctors, \( p \) increases without exception with increasing education, socioeconomic class, and believed personal network size and almost without exception with increasing age. A similar but weaker version of this occurs for Mailmen.

With regard to all six reference subpopulations, \( p \) increases with increasing believed personal network size without exception for Doctors and Quake Victims and almost without exception for Mailmen, Bus Drivers, TV Repairmen, and Priests. Thus, believed personal network size behaves like actual personal network size should behave with respect to

TABLE 6
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Occupation

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>home</th>
<th>oohm</th>
<th>retd</th>
<th>unem</th>
<th>stud</th>
<th>rsdl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.3986</td>
<td>.3906</td>
<td>.6957</td>
<td>.1948</td>
<td>.4196</td>
<td>.3391</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.1026</td>
<td>.1613</td>
<td>.0870</td>
<td>.1039</td>
<td>.1473</td>
<td>.1845</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.2243</td>
<td>.2575</td>
<td>.0870</td>
<td>.1948</td>
<td>.2455</td>
<td>.3734</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.1862</td>
<td>.2849</td>
<td>.2609</td>
<td>.2078</td>
<td>.2857</td>
<td>.3133</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.1432</td>
<td>.2651</td>
<td>.3043</td>
<td>.2078</td>
<td>.3750</td>
<td>.2575</td>
</tr>
<tr>
<td>Priests</td>
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<td>.2868</td>
<td>.4348</td>
<td>.2468</td>
<td>.2545</td>
<td>.2790</td>
</tr>
</tbody>
</table>
TABLE 7
Probabilities of Knowing Someone in the Event Subpopulation According to the Partition of the Sample by Believed Personal Network Size (in Hundreds)

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>no ans</th>
<th>&lt;1</th>
<th>1–5</th>
<th>5–10</th>
<th>10–15</th>
<th>&gt;15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>.3750</td>
<td>.2634</td>
<td>.3941</td>
<td>.4552</td>
<td>.5282</td>
<td>.5680</td>
</tr>
<tr>
<td>Mailmen</td>
<td>.0625</td>
<td>.1156</td>
<td>.1525</td>
<td>.1253</td>
<td>.1897</td>
<td>.2524</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>.0000</td>
<td>.2245</td>
<td>.2585</td>
<td>.2353</td>
<td>.2615</td>
<td>.4272</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>.3125</td>
<td>.1801</td>
<td>.2754</td>
<td>.3095</td>
<td>.3385</td>
<td>.3981</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>.3125</td>
<td>.1438</td>
<td>.2472</td>
<td>.3350</td>
<td>.4359</td>
<td>.4320</td>
</tr>
<tr>
<td>Priests</td>
<td>.1875</td>
<td>.2124</td>
<td>.2965</td>
<td>.2890</td>
<td>.3436</td>
<td>.4606</td>
</tr>
</tbody>
</table>

In order to see whether there is more information in this data, we plot the known value of 100e against the various values of p in Tables 3–7 for each of the six reference subpopulations. For each reference subpopulation this yields a range of p values, from the minimum to the maximum, which we call the p-spread for that subpopulation. These are plotted as horizontal lines in Fig. 1. On each p-spread is shown the full sample p value and corresponding α value from Table 2. Now, although there is no simple model curve of the form (12) which comes close to passing through all the points (p, e) given by the data in Table 2, we try to obtain the next best thing, namely, a simple model curve which intersects a maximum number of these p-spreads. Unfortunately, there is no such curve which intersects all six or even five of the six p-spreads. However, there is a small set of such curves which intersects the p-spreads for Doctors, Bus Drivers, and Quake Victims and comes close to intersecting the p-spreads for both Mailmen and TV Repairmen. The best estimating curve of this set, which just fails to intersect the p-spreads for Mailmen and TV Repairmen by about the same difference in p, has a nearest integer α value of 220. This value is virtually the same as that for the lines given by (19.7) and (17.8) and is easily within the bounds 235 ± 39 for the lines given by (16.7) and (18.7). This lends consistent support to the simple model with α = 220. We note that the p values where the curve for α = 220 intersects the p-spreads for Doctors, Bus Drivers, and Quake Victims are nonunique weighted combinations of the p values for various combinations of the informant subsamples given in Tables 3–7. Each such weighted combination of subsample p values (e.g., .35 · p(age 35–49) + .20 · p(13–16 years education) + .45 · p(middle class)) may thus be viewed as an indicator combination for the corresponding event subpopulation. The important question here is whether an indicator combination gives consistent results under repeated sampling.

From the number of respondents in each subclass with respect to believed personal network size (except No Answer), using the midpoint value as the mean for each of the first four size classes and 1800 for the mean of the last size class, we obtain the average believed person network size of 516. This is modestly larger than the values of α obtained relative to quake victims in both Mexican surveys.

We note that for no subclass in Tables 3–7 does the monotonicity relation between p and e given by (6) and (7) occur even approximately, which tends to disconfirm the simple model in the absence of major errors in the data. It appears that developing from this an accurate and precise method for estimating the size of an unknown subpopulation will require further development of the model and perhaps improvement of the data.

APPLICATION TO ANOTHER EVENT SUBPOPULATION IN MEXICO CITY

We can attempt to estimate the unknown size of the subpopulation of Rape Victims in Mexico City proper. In the second survey, for knowing
all six reference subpopulations. Except for believed personal network
size, $p$ decreases without exception for Bus Drivers with increasing so-
ioeconomic class and almost without exception with increasing age and
education. For Quake Victims and TV Repairmen the patterns of how
$p$ varies with increasing values of the four table variables are very similar.
For socioeconomic class and education the patterns of how $p$ varies for
the six reference subpopulations are also very similar, indicating that
functionally, for our purposes here, these are very similar attributes.

In order to see whether there is more information in this data, we plot
the known value of 100e against the various values of $p$ in Tables 3–7
for each of the six reference subpopulations. For each reference sub-
population this yields a range of $p$ values, from the minimum to the maximum,
which we call the $p$-spread for that subpopulation. These are plotted as
horizontal lines in Fig. 1. On each $p$-spread is shown the full sample $p$
value and corresponding $\alpha$ value from Table 2. Now, although there is
no simple model curve of the form (12) which comes close to passing
through all the points $(p, e)$ given by the data in Table 2, we try to obtain
the next best thing, namely, a simple model curve which intersects a
maximum number of these $p$-spreads. Unfortunately, there is no such
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curve of this set, which just fails to intersect the $p$-spreads for Mailmen
and TV Repairmen by about the same difference in $p$, has a nearest integer $\alpha$ value of 220. This value is virtually the same as that for the
lines given by (19.r) and (17.s) and is easily within the bounds $235 \pm 39$
for the lines given by (16.r) and (18.r). This lends consistent support to
the simple model with $\alpha \approx 220$. We note that the $p$ values where the
curve for $\alpha = 220$ intersects the $p$-spreads for Doctors, Bus Drivers, and
Quake Victims are nonunique weighted combinations of the $p$ values
for various combinations of the informant subsamples given in Tables
3–7. Each such weighted combination of subsample $p$ values (e.g.,

\[ 0.35 \cdot p_{(\text{age 35-49})} + 0.20 \cdot p_{(13-16 \text{ y educ})} + 0.45 \cdot p_{(\text{middle class})} \]

may thus be viewed as an indicator combination for the corresponding event
subpopulation. The important question here is whether an indicator com-
bination gives consistent results under repeated sampling.

From the number of respondents in each subclass with respect to believed
personal network size (except No Answer), using the midpoint value as the mean for each of the first four size classes and 1800 for the
mean of the last size class, we obtain the average believed person network
size of 516. This is modestly larger than the values of $\alpha$ obtained relative
to quake victims in both Mexican surveys.

We note that for no subclass in Tables 3–7 does the monotonicity
relation between $p$ and $e$ given by (6) and (7) occur even approximately,
which tends to disconfirm the simple model in the absence of major errors
in the data. It appears that developing from this an accurate and precise
method for estimating the size of an unknown subpopulation will require
further development of the model and perhaps improvement of the data.

APPLICATION TO ANOTHER EVENT SUBPOPULATION
IN MEXICO CITY

We can attempt to estimate the unknown size of the subpopulation of
Rape Victims in Mexico City proper. In the second survey, for knowing

<table>
<thead>
<tr>
<th>Subpopulation</th>
<th>no ans</th>
<th>&lt;=1</th>
<th>1–5</th>
<th>5–10</th>
<th>10–15</th>
<th>&gt;=15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctors</td>
<td>0.3750</td>
<td>0.2634</td>
<td>0.3941</td>
<td>0.4552</td>
<td>0.5282</td>
<td>0.6800</td>
</tr>
<tr>
<td>Mailmen</td>
<td>0.0625</td>
<td>0.1156</td>
<td>0.1525</td>
<td>0.1253</td>
<td>0.1897</td>
<td>0.2524</td>
</tr>
<tr>
<td>Bus Drivers</td>
<td>0.0000</td>
<td>0.2245</td>
<td>0.2585</td>
<td>0.2353</td>
<td>0.2615</td>
<td>0.4272</td>
</tr>
<tr>
<td>Quake Victims</td>
<td>0.3125</td>
<td>0.1801</td>
<td>0.2754</td>
<td>0.3095</td>
<td>0.3385</td>
<td>0.3981</td>
</tr>
<tr>
<td>TV Repairmen</td>
<td>0.3125</td>
<td>0.1438</td>
<td>0.2472</td>
<td>0.3350</td>
<td>0.4359</td>
<td>0.4320</td>
</tr>
<tr>
<td>Priests</td>
<td>0.1875</td>
<td>0.2124</td>
<td>0.2995</td>
<td>0.2890</td>
<td>0.3436</td>
<td>0.4006</td>
</tr>
</tbody>
</table>
TABLE 8
Estimates of the Number of Rape Victims in Mexico City for Various Obtained Values of \( \alpha \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>116</th>
<th>196</th>
<th>220</th>
<th>235</th>
<th>274</th>
<th>332</th>
<th>516</th>
<th>810</th>
<th>2254</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>14,883</td>
<td>8811</td>
<td>7850</td>
<td>7349</td>
<td>6303</td>
<td>5202</td>
<td>3348</td>
<td>2133</td>
<td>766</td>
</tr>
</tbody>
</table>

a rape victim we obtained the estimate \( p = 0.1491 \) with \( t = 10,700,000 \).
For the various \( \alpha \) values of interest obtained earlier, we obtain the estimates of the subpopulation of Rape Victims given in Table 8.

From the curve for \( \alpha = 220 \) in Fig. 1 we can estimate bounds for the number of Rape Victims in Mexico City proper according to \( \min(p) = 0.0571 \) (for Age > 65 years) and \( \max(p) = 0.3111 \) (for Education > 16 years). When the resulting \( p \)-spread for Rape Victims intersects this curve at minimum \( p \), \( e \) is minimum, and when it intersects it at maximum \( p \), \( e \) is maximum. As generally shown by the figure, we obtain \( \min(e) = 0.000267 \) and \( \max(e) = 0.001692 \), which translates to \( \min(e) = 2859 \) and \( \max(e) = 18110 \), so \( 2859 \leq e \leq 18110 \). This range is too large for estimating \( e \), but note that all \( e \) values in Table 8 except those determined by \( \alpha = 810 \) and 2254 lie in this interval. For the \( \alpha \) values lying in the previously estimated interval 235 ± 39 or 196 ≤ \( \alpha \) ≤ 274 we have 6303 ≤ \( e \) ≤ 8811, which is a considerably better estimate.

APPLICATION TO AN EVENT SUBPOPULATION IN THE U.S.

In a Media General—Associated Press poll of 1304 randomly selected adults across the U.S. taken in April 1987, one of the questions asked was whether the respondent knew anyone with AIDS (cf. Kilman, 1987). Seven percent of them said they did. Using this figure, an estimated May 1, 1987 U.S. adult population (over 17 years of age) of 179,955,000 based on data from the U.S. Bureau of the Census (1987a,b), and the diagnosed number of AIDS victims as of early May 1987 of 35219, we can apply the simple model to estimate the corresponding value \( \alpha = 371 \). For the maximum error of ±0.27 in \( p \) (at an assumed confidence level of 95%), this gives a range of 222 ≤ \( \alpha \) ≤ 523. This range is consistent with values and bounds for \( \alpha \) which we have determined with this model for the Federal District of Mexico City and for Mexico City proper. Because of the cultural differences between the U.S. and Mexico, however, the similarity of these \( \alpha \) values to those for Mexico City may only be coincidental.

REFERENCES


TABLE 8
Estimates of the Number of Rape Victims in Mexico City for Various Obtained Values of $\alpha$

<table>
<thead>
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<th>196</th>
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<th>235</th>
<th>274</th>
<th>332</th>
<th>516</th>
<th>810</th>
<th>2254</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>14,883</td>
<td>8811</td>
<td>7850</td>
<td>7349</td>
<td>6033</td>
<td>5202</td>
<td>3348</td>
<td>2133</td>
<td>766</td>
</tr>
</tbody>
</table>

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